PHYSICAL REVIEW
LETTERS

VOLUME 9 OCTOBER 1, 1962 NUMBER 7

# THEORY OF SECOND SOUND ABSORPTION IN ROTATING HELIUM*

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(Received September 13, 1962)

An alternative explanation is proposed for the additional second sound attenuation observed experimentally[1] in rotating liquid helium II. Such possibility appears significant in view of the implications attached to the earlier treatment by Hall and Vinen,[2] who attribute loss to friction of quantized vortices slicing through normal fluid. By tacitly relating the process to vorticity, their interpretation has become regarded as a first triumph of the Onsager-Feynman quantized vortex theory.[3,4] The present approach ignores the role of any vortex scheme in explaining absorption, but relies instead upon the over-all Feynman concept of rotating liquid helium. Losses are attributed to interactions between the second sound waves and the local, irrotational regions of superfluid implicit to that concept. By thus shifting the focus for the origin of absorption the observed behavior may be derived directly, without introducing fundamental physical constants or reliance upon undetermined or adjustable vortex parameters.

The treatment requires one assumption regarding the nature of rotating liquid helium, the existence (following Feynman) of local regions or domains within which the condition $\nabla \times v_s = 0$ holds. Shape (nearly) and size of these domains remain unimportant, as does also the nature of the "flaws" or vorticity regions separating them. Loss of wave energy follows directly upon reconciling the background rotational character of liquid helium II with the second sound process, subject to the condition of minimum free energy. The adjustments in local energy density required by this constraint are interpreted as irreversible exchanges of energy from the wave system, produced as the result of forced conversion between fluid components. Energy thus dissipated appears ultimately as "stagnant" heating, in contrast to the originally ordered heat flow of the second sound phenomenon.

Treated most conveniently in coordinates rotating with the helium system at uniform angular velocity $\Omega$ (rad/sec), the equilibrium rotation condition then comprises "stationary" normal fluid component co-existing with superfluid component occupying "fixed location" domains. Within each domain the superfluid appears to perform relative "solid body" rotation in the reverse sense (angular velocity $-\Omega$ relative to these rotating coordinates). The basic contrast thus existing between the split velocity field characterizing rotation and the purely linear (and longitudinal[5]) split velocity field of temperature waves provides a special dilemma for second sound propagation in rotating helium.

Consider a second sound wave system of arbitrary form propagated at arbitrary direction through liquid helium rotated at uniform angular velocity $\Omega$ (rad/sec). At any point within the wave the associated kinetic energy density of the combined system, expressed with respect to the ro-

tating coordinate framework, is given by

$$\text{kinetic energy density} = \tfrac{1}{2}\rho_n v_n^2 + \tfrac{1}{2}\rho_s[\vec{v}_s + (\vec{r}\times\vec{\Omega})]^2 \quad (1)$$

(ergs/cc). Here symbols retain the usual significance, with $\rho_n,\rho_s$ and $v_n,v_s$ representing density (g/cc) and velocity (cm/sec) for normal and superfluid, respectively. Applicable to whichever "solid body" domain contains the point in question, the vector $\vec{r}$ represents the radial displacement from the corresponding rotation axis. Any interaction between the temperature wave and the background rotational state could evidently arise only through some coupling effect related to the cross-term $\rho_s[\vec{v}_s\cdot(\vec{r}\times\vec{\Omega})]$ for superflow in Eq. (1). We note, however, that the over-all kinetic energy content of the system remains unaffected by this "coupling term" as integration over any composite domain region immediately reveals.

Although the asymmetry produced in the coupling term by $\vec{r}$ precludes any direct effect on system kinetic energy, associated higher order properties are not similarly limited. Specifically, a second-order[6] Bernoulli-type pressure distribution develops in the same asymmetric pattern, produced also by internal convection and given in fact by the negative of (1). But the associated potential energy field is quadratic[7] in pressure and therefore constitutes a symmetric function (second order in coupling term and fourth order in particle motion). Integration of this potential energy over the domain area necessarily exceeds zero, and accordingly the system free energy appears increased through interaction between second sound and helium rotational background.

Clearly an energetically more favorable situation could result naturally if such coupling potential could be precluded without affecting the kinetic energy. The only relaxable constraint offered being the relative fluid concentration, the question arises whether local adjustments between fluid components[8] might thus lower the system free energy. The postulate is therefore presented here that the principle of minimum free energy requires an adjustment between component fluid densities just sufficient to keep the resultant kinetic energy density independent of local rotational behavior, and that the losses of ordered kinetic energy attendant on such processes degenerate to stagnant heating and thereby constitute absorption of temperature waves. Minute portions of either component reversing direction under forced conversion to flow conditions of the other necessarily dissipate[9] wave energy.

Applied to Eq. (1), the above postulate implies adjustments in wave kinetic energy $\Delta W$ (erg/cc) given by

$$\Delta W = \Delta[\tfrac{1}{2}\rho_n v_n^2 + \tfrac{1}{2}\rho_s v_s^2] = \rho_n[\vec{v}_n\cdot(\vec{r}\times\vec{\Omega})], \quad (2)$$

where the zero momentum condition $\rho_n v_n + \rho_s v_s = 0$ has been used. Introducing the relative (closing) speed $|\vec{v}_n - \vec{v}_s|$ between components, the time rate of delivery,[10] as irreversible transfer, of wave energy from the system becomes

$$\begin{aligned} dW/dt &= -\rho_n(|\vec{v}_n - \vec{v}_s|)|\vec{v}_n\cdot(\vec{r}\times\vec{\Omega})| \\ &= -\rho(\rho_n/\rho_s)(|\vec{v}_n|)|\vec{v}_n\cdot(\vec{r}\times\vec{\Omega})| \\ &= -(2W/c_2)|\vec{c}_2\cdot(\vec{r}\times\vec{\Omega})|, \end{aligned} \quad (3)$$

where the kinetic energy density $[\tfrac{1}{2}\rho(\rho_n/\rho_s)v_n^2]$ and wave velocity $\vec{c}_2$ (cm/sec), respectively, for second sound have been introduced. Space prohibits details here but, following integration across a domain midsection and normalization by domain area (square for example), the equation of attrition[11] for the average wave intensity $\gamma$ (erg/sec-cm$^2$) along the propagation axis $y$ becomes

$$d\gamma/\gamma = -(|\Omega|/|c_2|)\sin\theta\, dy, \quad (4)$$

as a function of angle $\theta$ between the axis of rotation and the propagation vector.

Finally the amplitude attenuation factor $\alpha$ (cm$^{-1}$)

$$\alpha = (\Omega/2c_2)\sin\theta \quad \text{(square domains)} \quad (5a)$$

$$= (4/\pi)(\Omega/2c_2)\sin\theta \quad \text{(circular domains)}, \quad (5b)$$

results[12] for two limiting domain shapes. These agree closely with the experimental observations of Hall and Vinen[1] whose measurements showed $\alpha$'s of value effectively (5a) for temperatures above 1.5°K and averaging about (5b) within 1.2°K-1.5°K. The results appear well bracketed by

$$(\Omega/2c_2)\sin\theta < \alpha < (4/\pi)(\Omega/2c_2)\sin\theta.$$

All the unusual characteristics exhibited by second sound attenuation in rotating liquid helium are thereby accounted for, including the following: (1) nondependence on wave frequency $(\omega/2\pi)$, (2) reciprocal dependence on $c_2$ (in contrast to usual reciprocal quadratic dependence for wave absorption), and (3) maximum absorption for propagation perpendicular to the rotation axis and minimum absorption along the rotation axis. In contrast to reference (2), the present treat-

ment requires introducing neither Planck's constant ($\hbar$), the mass of the helium atom ($m$), nor the inner or outer undetermined radii ($a$ and $b$) of quantized vortices. Besides relying thus upon less profound physical concepts, the result requires no adjustment to experiment.

*Work supported jointly by the Alfred P. Sloan Foundation and the National Science Foundation.

[1]H. Hall and W. Vinen, Proc. Roy. Soc. (London) A238, 204 (1956).

[2]H. Hall and W. Vinen, Proc. Roy. Soc. (London) A238, 215 (1956).

[3]L. Onsager, Suppl. Nuovo cimento 2, 249 (1949).

[4]R. Feynman, Progress in Low-Temperature Physics, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. 1, Chap. II, p. 36.

[5]Additional dynamic complications for rotating systems (Coriolis effects) are neglected as higher order in $\Omega$ for absorption (quadratic).

[6]Recalling that temperature waves support no first-order pressure fluctuations, we recognize that second-order processes assume primary importance.

[7]Local density deviations from the ambient contribute positively to the potential energy, regardless of sign (consider the potential energy content of ordinary sound).

[8]In contrast to the normally adiabatic nature of second sound, involving no actual conversion between phases.

[9]Upon conversion from superfluid, for example, there results a small portion of normal fluid travelling in the wrong direction, with consequent frictional retardation and degeneration of kinetic energy. The reverse process presents greater subtleties.

[10]Note that whether in "closing" the normal component enters regions of greater or less wave energy remains immaterial, as either process produces irreversible energy transfer (thus the absolute value signs). Such a consideration also enters integration of Eq. (4), following.

[11]Note independence on domain size upon normalization for density calculation.

[12]The present result remains valid only for conditions wherein the wavelength greatly exceeds the domain size ($\lambda \gg a$). A re-evaluation would be necessary for the opposite extreme, and in fact, for high enough frequencies ($\lambda \ll a$), the effect may conceivably disappear.

## TUNNELING-ASSISTED PHOTON EMISSION IN GALLIUM ARSENIDE *pn* JUNCTIONS

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(Received July 2, 1962; revised manuscript received August 20, 1962)

When a gallium arsenide *pn* junction is biased in the forward direction with a voltage nearly equal to the energy gap, efficient radiative recombination is observed. We shall show evidence that this recombination involves tunneling of carriers across the space charge region of the *pn* junction.

The mechanism is believed to consist of an electron from the conduction band of the *n*-type region tunneling to a virtual state (in the energy gap) from which it makes a radiative transition to the valence band of the *p*-type region, thus recombining with a hole.

The possibility that photon-assisted tunneling may occur in germanium tunnel diodes has been proposed by Aigrain[1] and by Sommers,[2] but, to the best of our knowledge, no clear-cut evidence for this mechanism has been reported.[3] However, the converse effect, field-assisted absorption of photons,[4] has been observed in a number of materials[5] including GaAs.[6]

For our experiment, GaAs *pn* junctions were made between degenerately doped regions either by alloying a tin dot to a zinc-doped crystal or by diffusing zinc into a degenerate *n*-type crystal. The diodes were operated at liquid helium temperature. The forward current-vs-voltage characteristic of these diodes shows that when the applied voltage is nearly equal to the energy gap (about 1.5 eV at 4.2°K), the current rises very rapidly with increasing voltage. It is under this bias condition that an infrared emission of near band-gap energy is obtained. The emission spectrum, which has the form of Fig. 1, peaks at an energy somewhat lower than the gap, and shifts to higher energies as the current increases. A shift from 1.450 to 1.479 eV was obtained with a change in current density from about 70 to 1000 A/cm$^2$.

The exact value of the energy gap is not known because the material is heavily doped. However, previously reported measurements[7] of photovoltaic threshold, absorption edge, and V-I characteristics (voltage at which the current rises rapidly)

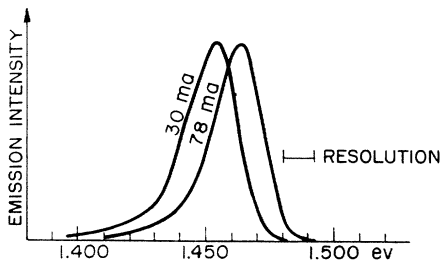


FIG. 1. Emission spectra of a GaAs *pn* junction at 4.2°K for two currents through the junction.